Evolutionary Computation

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Master Artificial Intelligence

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GA: mutation possiblities

gene mutation:
: just change randomly one (or more) gen(es) to another permitted allele

gene flip:
: interchange the values of two (or more) genes

gene sequence displacement:
: cut a sequence/part and insert at another position

gene sequence inversion:
: revert the order of a (partial) sequence

what-ever-you-like:
: do something, be happy...

GA: mutation rules of thumb

- The mutation rate should be, more or less, inversely proportional to the size of the genome.
- For larger populations maybe reduce mutation rate in the on-going optimization process.
- With low diversity in the populations, maybe a larger mutation rate helps.

GA: crossover possiblities

simple crossover:

parents	cut		children
(101101)	(10|1101)	$\longrightarrow$	(100111)
(010111)	(01|0111)	$\longrightarrow$	(011101)
$(2, 8, 98, 3)$	$(2, 8, \vert 98, 3)$	$\longrightarrow$	$(2, 8, 40, 4)$
$(1, 9, 40, 4)$	$(1, 9, \vert 40, 4)$	$\longrightarrow$	$(1, 9, 98, 3)$

k-point crossover:

cut at *k* points and interchange the corresponding parts (variation: take *k* at random)

uniform crossover:

interchange each gene with certain probability

multiple parent mating:

use more then two parents and interchange genes (variation: merge entire parent set)

GA: crossover possiblities (continued)

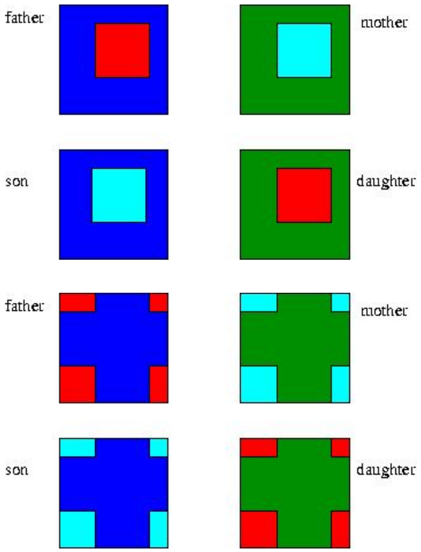
arithmetic crossover: assign to children convex combination of parent genes with some random weight, $\alpha \in [0,1]$, e.g., with $\alpha = 0.7$ on second gene:
$34.5 \cdot 0.7 + 13.5 \cdot 0.3 = 28.2$
$(1.23, 34.5, -2.1) \longrightarrow (1.23, 28.2, -2.1)$
$13.5 \cdot 0.7 + 34.5 \cdot 0.3 = 19.8$
$(10.5, 13.5, 23.1) \longrightarrow (10.5, 19.8, 23.1)$
(again variations: as *k*-point, or with all genes, or with *k* at random)

blended crossover: blend two corresponding parent genes with a certain, usually fixed, value $\alpha \in [-0.5, \infty]$ according to the current gene spread

simulated binary crossover: blend two corresponding parent genes according to a suitable probability density function

what-ever-you like: remember, do something, be happy...

GA: crossover example (2-point cyclic crossover)



- select two grid points
- interchange rectangles

GA: principal loop

A genetic algorithm can be summarized in the following principal loop (parts that are dealt with marked with *DONE*):

```
InitializePopulation()      # initialization
EvaluateIndividuals()       # evaluation        DONE
while not Stopping():       # stopping
  DetermineParents()          # selection
  GenerateChildren()          # recombination   DONE
  MutateChildren()            # mutation        DONE
  EvaluateIndividuals()       # evaluation      DONE
  ReestablishPopulation()     # selection
```

GA: selection

- In the principal loop, there are two selection processes:
  - How to select the parents to generate the off-springs? and
  - How to rearrange the final population or next generation?
- We use the following notation:
  - μ stands for the number of individuals in the population
  - λ stands for the number of children being generated
- We distinguish two main strategies:
  - $(\mu+\lambda)$-strategy
    - from the μ individuals of the current generation select the parents and generate λ children
    - from the $\mu+\lambda$ individuals choose the μ best ones as new generation
  - (μ,λ)-strategy
    - from the μ individuals of the current generation select the parents and generate $\lambda \geq \mu$ children
    - from the λ children choose the μ best ones as new generation

The second question from above is answered.

GA: selection options

To select the parents being allowed to have off-springs, there exists a bunch of suggestions:

roulette wheel:	assign to each individual a fraction of the wheel according to its relative fitness and spin the wheel (variation: smooth, weight, or normalize the fitness somehow, e.g., use $\log$ of objective function, or use z-score)
rank based:	order the individuals according to fitness and select with a probability weighted by the rank (variation: compute selection probability with linear function of rank, so the least ranked still gets certain probability to get selected)

GA: selection options (continued)

tournament based: draw a certain number of random individuals, select the best one as parent (variation: select directly the best two as parents)

truncation selection: only the individuals with highest fitness values will be parents

what-ever-you like: remember, do something, be happy...

The first question (from two slides earlier) is answered.

GA: initialization

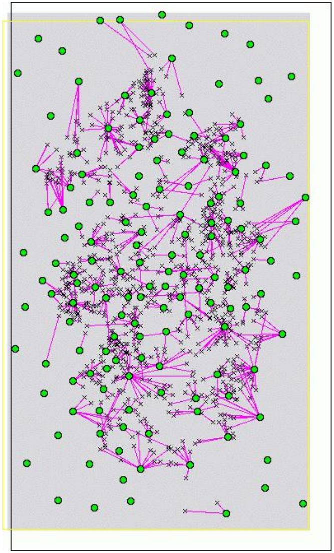
- generate the initial population with random genomes
    - take into account that the distribution according to genotype not necessarily is similar to the distribution in phenotype
    - maybe there are many individuals with very low fitness
    - the initial convergence rate might be slow
- generate the initial population with individuals from another heuristic algorithm or various such algorithms
    - the population might be biased into a certain region of the search space
    - the diversity of the population might be low
    - the convergence rate might be trapped early in a local optimum
- recommendation: use a mixture of both

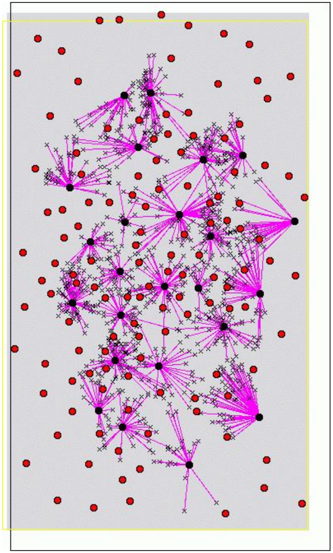
GA: termination criteria

There are many possibilities when to stop the iteration of a genetic algorithm:

- once the first solution has been found
- once a sufficiently good solution has been found
- once the optimum has been found
- once a certain number of iterations has been executed
- once the diversity of the population is below a certain threshold
- once the convergence rate of the improvement is below a certain threshold
- once a certain amount of runtime has been spent
- recommendation: use an or-mixture of all

GA: results for the example (2007)

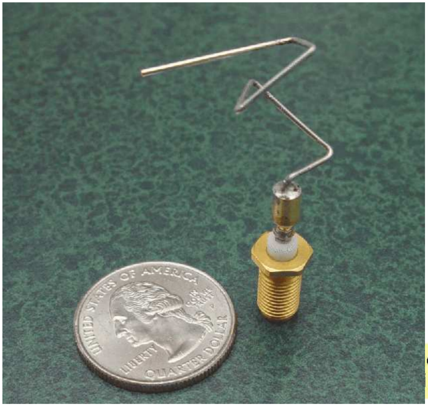

119 of 149 nodes used

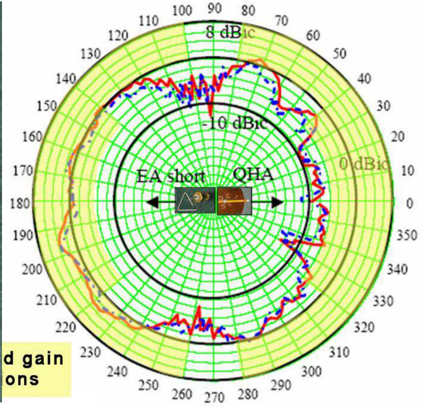

24 of 149 nodes used

GA: use for antenna design (small satellites)

Horny, AlGlobus, Linden, Lohn: Automated Antenna Design with Evolutionary Algorithms (2012)

`https://arc.aiaa.org/doi/10.2514/6.2006-7242` or `https://www.researchgate.net/publication/228909002_Automated_Antenna_Design_with_Evolutionary_Algorithms`





GA: diversity

- The diversity measures, in some sense, the non-similarity between the individuals of a population.
- E.g., Hamming-distance over the bitstring (using exor): $010010 \otimes 101000 = 111010 \longrightarrow 4$
- e.g., delta-distance over the integer (or real) sequence: $\sum_i |x_i - x'_i|$ (being x and x' two individuals)
- There are much more similarity measures

  https://towardsdatascience.com/

  17-types-of-similarity-and-dissimilarity-measures-used-in-data-science-3eb914d2681/.
- Similar individuals in a population reduce the diversity and the genetic algorithm maybe gets stuck in some region of the search space (maybe, but not necessarily, a local minimum).

GA: diversity

- To augment the diversity, we have only the mutation operation, provided the mutation becomes visible in the next generation.
  (Observe: whenever an allele disappears in a population, most of the crossover operations cannot regenerate it!)
- Another possibility is just to regenerate a completely or partially new population.

GA: elitism

We have to draw the decision whether the best individual(s) is (are) forced to belong unmodified to the next generation.

- Elitism might help to converge faster.
- Elitism might reduce diversity faster.
- The consequences of this trade-off are problem dependent.

GA: final remarks

- The difficulties of understanding and analizing genetic algorithms lie in the fact that they implement a combination of random search (by mutation) and biased search (by recombination).
- Genetic algorithms need unique and problem-specific mutation and recombination operators, which makes it more challenging to implement a generic version that can be easily applied to different optimization problems.
- Nature still has its somewhat better approach: DNA, RNA, gene expression with regulation, proteins, and mitochondria (mtDNA), and, and, and,...
(e.g. https://www.ncbi.nlm.nih.gov/books/NBK459456/)

evolutionary programming (EP)

Once we have seen GA, evolutionary programming is somewhat simpler: it just uses mutation.

- there exist only the phenotypes, let's say x_i (for $i = 1, \ldots, n$), i.e., n individuals in the population
- modification (mutation) is realized over the phenotypes as:

$$x_i' = x_i + r_i \sqrt{\beta f(x_i) + \gamma}$$

  being $\beta > 0$ and $\gamma \geq 0$ tuning parameters (for instance $\beta = 1$ and $\gamma = 0$) and r_i is a random value taken from a normal distribution with mean 0 and variance 1 (i.e., $r_i \in N[0,1]^n$).
- Note that the fitness (objective function f) must be shifted, so the minimum is positive.
- Usually a $(\mu + \mu)$-selection strategy is used: all individuals are mutated and the best μ individuals are kept.

EP: principal loop

A evolutionary programming algorithm can be summarized in the following principal loop:

```
InitializePopulation()
EvaluateIndividuals()
while not Stopping():
  GenerateChildrenByMutation()
  EvaluateIndividuals()
  ReestablishPopulation()
```

differential evolution (DE)

Once we have seen GA, differential evolution is somewhat simpler: it just uses a special type of recombination (crossover).

- there exist only the phenotypes, let's say x_i (for $i = 1, \dots, n$), i.e., n individuals in the population
- For each individual we select three other individuals, say x_j, x_k, x_l, to compute a mutant vector v_i

$$v_i = x_j + F \cdot (x_k - x_l)$$

  being $F \in [0.4, 0.9]$ (usually) a tuning parameter.
- Then we generate an off-spring with a uniform crossover between individual x_i and mutant v_i using a certain threshold c
- Usually a $(\mu + \mu)$-like selection strategy is used: all individuals are used to generate off-springs, and the best are kept.

DE: principal loop

A differential evolution algorithm can be summarized in the following principal loop:

```
InitializePopulation()
EvaluateIndividuals()
while not Stopping():
  GenerateChildrenByDiffusion()
  EvaluateIndividuals()
  ReestablishPopulation()
```

DE: some variations

- One might consider to use always the best individual found so far as individual x_j.
- The tuning parameter F might vary, i.e., taking the value from a uniform or a normal distribution.
- One might use DE on discrete sets as well by just rounding the mutants appropriately (or search in the close integer neighborhood according to the dimension of the underlying problem).
- (My opinion) Differential evolution is not just a genetic algorithm, as there is no genotype, rather the other way round: a genetic algorithm using the phenotype as genotype, no mutation, and a random recombination, becomes a differential evolution algorithm.

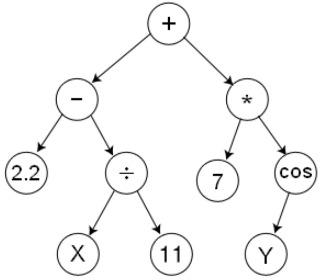
genetic programming (GP)

Once we have seen GA, genetic programming is a genetic algorithm with some special phenotypes and genotypes.

- the genotype is a (simple) program described as a syntax tree that can be written as well with Polish notation (prefix notation), see next slide...
- the parenthesis can be eliminated, interpretation of the corresponding expression is easy to perform with a stack automaton.
- some properties of the execution of the resulting program (as phenotype) are used as fitness (see example, later)

GP: syntax tree

syntax tree and Polish notation



$$\left(2.2 - \left(\frac{X}{11}\right)\right) + \left(7 * \cos(Y)\right)$$

- `(2.2-(x/11))+(7*cos(y))`
- `(+ (- (2.2 (/ X 11))) (* (7 cos(Y))))`
- `+ - 2.2 / X 11 * 7 cos Y`

image taken from wikipedia

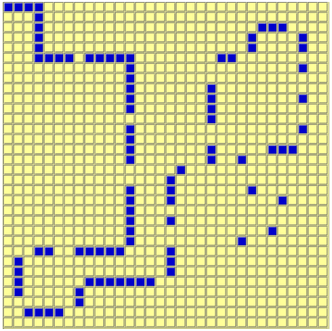
GP: mutation and crossover operations

- the programs are modified with adecuate mutation and crossover operations
- mutation:
  - change a node, but take care to keep a valid syntax tree (maybe subtrees must be removed or added)
  - rotate nodes
  - interchange nodes
- crossover: interchange a subtree of one parent with a subtree of the other parent

GP: example

Program a robot (ant) that starts at some cell (usually a corner) and tries to find as many objects (food) with as few steps as possible.

Santa Fe Trail



nodes: turn-left, turn-right, move, if-food-ahead